ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/1

PAPER 1

NOVEMBER 2022 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF7
- Scientific calculator (Non-programmable)

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

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[Turn over

1 (i) Expand $(4-3x)^{-\frac{1}{2}}$ up to and including the term in x^3. [4]

(ii) State the values of x for which the expansion is valid. [1]

2 The functions h and g are defined by $h: x \longrightarrow x^3 \; x \in \mathbb{R}$
$g: x \longrightarrow \sqrt{x} \; x \geq p$

(i) State the least value of p for which g is defined. [1]

(ii) find in the form x^n, where n is rational

(a) gh, [2]

(b) g^2. [2]

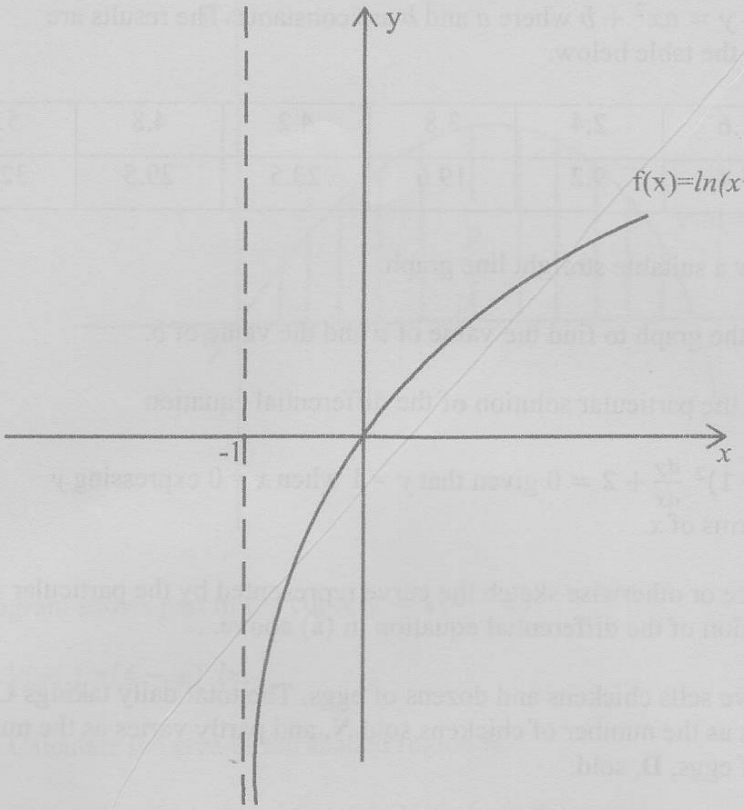
3 A curve **C** is defined by the parameric equations:

$x = 3 + 4Cos\theta$
$y = 1 + 4Sin\theta.$

(i) Show that the equation of **C** represents a circle. [3]

(ii) State the centre and radius of the circle. [2]

4 The diagram below shows the graph of $y = f(x)$.



On separate diagrams sketch the graphs of

(a) $y = |f(x)|$, [2]

(b) $y = f(x - 2)$, [2]

(c) $y = 1 - f(x)$. [2]

5 In a particular experiment the two variables x and y were found to satisfy the equation $y = ax^2 + b$ where a and b are constants. The results are displayed in the table below.

x	1.6	2.4	3.8	4.2	4.8	5.0
y	5.4	9.2	19.6	23.5	29.5	32.3

(a) Draw a suitable straight line graph. [4]

(b) Use the graph to find the value of a and the value of b. [3]

6 **(a)** Find the particular solution of the differential equation

$(x+1)^2 \frac{dy}{dx} + 2 = 0$ given that $y = 1$ when $x = 0$ expressing y in terms of x. [4]

(b) Hence or otherwise sketch the curve represented by the particular solution of the differential equation in **(a)** above. [3]

7 A cooperative sells chickens and dozens of eggs. The total daily takings **C**, partly varies as the number of chickens sold **N**, and partly varies as the number of dozens of eggs, **D**, sold.

(a) Write down an expression for **C** in terms of **N**, **D** and the constants h and k. [1]

(b) A total of \$120 was collected when 15 chickens and 15 dozens of eggs were sold and a total of \$90 was collected when 10 chickens and 20 dozens of eggs were sold.

Find the value of h and the value of k. [3]

(c) State what the values of h and k represent in this situation. [1]

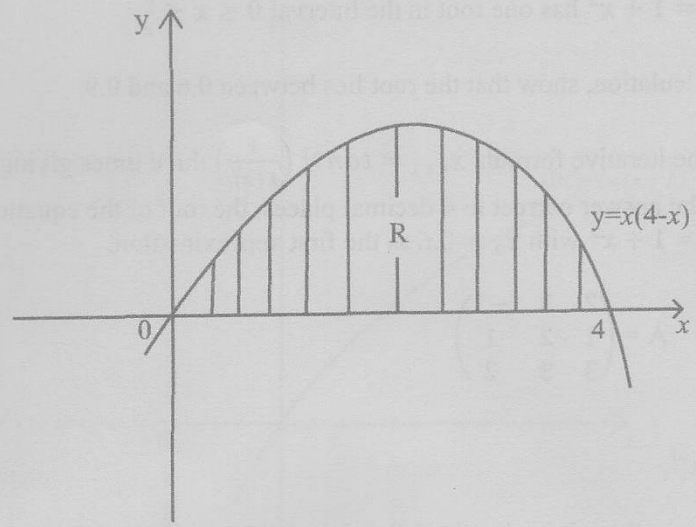
(d) Find the total daily takings if 20 chickens and 30 dozen eggs were sold. [2]

8 Prove by Mathematical induction that $\sum_{r=1}^{n} 8r - 1 = 4n^2 + 3n$ for $n \in \mathbb{N}$. [7]

9 **(a)** Express $\frac{2}{4r^2-1}$ in partial fractions. [3]

(b) Hence or otherwise show that $\sum_{r=1}^{n} \frac{2}{4r^2-1} = \frac{2n}{2n+1}$. [6]

10



The diagram shows part of the curve $y = x(4 - x)$.

(a) Find $\int x(4 - x)\, dx$. [2]

(b) Calculate the area of the shaded region R. [2]

(c) The region R is rotated through 360° about the x-axis.

Calculate the exact volume of the solid generated in terms of π. [5]

11 **(a)** Find the range of values of k for which the equation $x^2 + (3k - 2)x + k + 2 = 0$ has no real roots. [5]

(b) Solve the equation $2x^3 - 9x^2 + 7x + 6 = 0$. [5]

12 $f(x) = ax^4 - 9x^3 + bx^2 - 3x + 2$ is exactly divisible by $x + 2$ and leaves a remainder -12 when divided by $x - 1$.

(a) Find the values of a and b. [4]

(b) By using the values of a and b.

(i) verify that $1 - 2x$ is a factor. [2]

(ii) show that $f(x)$ has only 2 real roots. [4]

13 **(a)** By sketching a suitable pair of graphs show that the equation

$cotx = 1 + x^2$ has one root in the interval $0 \leq x \leq \frac{\pi}{2}$. [3]

(b) By calculation, show that the root lies between 0,6 and 0,9. [3]

(c) Use the iterative formula $x_{n+1} = tan^{-1}\left(\frac{1}{1+x_n^2}\right)$ three times giving in each case the answer correct to 4 decimal places, the root of the equation $cotx = 1 + x^2$ with $x_0 = 0.6$ as the first approximation. [4]

14 Given that $A = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ 3 & 3 & 2 \end{pmatrix}$

(a) find A^{-1}, [7]

(b) hence or otherwise solve the following simultaneous equations:

$$2x + 3y - z = 0$$
$$x + 2y + z = 2$$
$$3x + 3y + 2z = 4$$ [4]

15 The points **A**, **B** and **C** have position vectors given respectively by $\boldsymbol{a} = 3\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ $\boldsymbol{b} = 2\boldsymbol{i} + 5\boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{c} = 3\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k}$.

(a) Find

(i) $A\hat{B}C$, [3]

(ii) the area of the triangle **ABC** in exact form, [3]

(iii) the position vector of **F**, the midpoint of **AB**. [3]

(b) Hence or otherwise show that **CF** is perpendicular to **AB**. [3]