ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/2

PAPER 2

NOVEMBER 2022 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF7
- Scientific calculator (Non-programmable)

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in **Section A** and any **five** questions from **Section B**.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

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[Turn over

Section A [40 marks]

Answer **all** questions in this section.

1 The point **A** has coordinates (3,4). The lines l_1 with equation $2y - x - 10 = 0$ and l_2 with equation $y - 3x = 0$ intersect at **B**.

(a) Find the coordinates of **B**. [2]

(b) Hence or otherwise show that **AB** is perpendicular to $2y - x - 10 = 0$. [3]

2 The total cost, C, in dollars of hotel services is partly constant and partly varies with the cost of food, F, consumed by an individual per day. When **C** = 53, **F** = 10 and when **C** = 68, F = 15.

(a) Form an equation connecting **C** and **F**. [4]

(b) Find the total cost C, when the cost of food consumed by an individual on a certain day costs \$71,25. [2]

3 **(a)** Show that $\tan 4\theta \equiv \frac{4t(1-t^2)}{1-6t^2+t^4}$ where $t = tan\theta$. [4]

(b) Hence or otherwise prove that when $tan\theta = \frac{1}{5}$, $\tan\left(4\theta - \frac{1\pi}{4}\right) = \frac{1}{239}$. [4]

4 The eleventh term of an arithmetic progression is 33. The first, third and ninth terms of the arithmetic progression form consecutive terms of a geometric progression.

Find the

(a) first term and common difference of the arithmetic progression, [6]

(b) sum of the first 10 terms of the geometric progression. [3]

5 Operation $\Diamond$ is defined on the set {T, H, O, E} as shown in the table below.

$\Diamond$	T	H	O	E
T	O	H	T	E
H	E	T	H	O
O	T	H	O	E
E	H	O	E	T

(a) State the value of T$\Diamond$H. [1]

(b) State whether the operation is commutative and why. [2]

(c) **(i)** Name the identity element. [1]

(ii) Justify your answer. [1]

(d) Find the inverse of each element. [4]

(e) T $\Diamond$ (H $\Diamond$ O) = (T $\Diamond$ H) $\Diamond$O.

(i) Is this true or false? [1]

(ii) Explain the answer. [2]

Section B [80 marks]

Answer any **five** questions from this section.

Each question carries 16 marks.

6 The parametric equation of a curve are $x = 3\text{Cos}2\theta$ and $y = -2\text{sin}2\theta$ where θ is a parameter.

(a) **(i)** Find $\frac{dy}{dx}$ in terms of θ, simplifying the answer. [2]

(ii) Hence or otherwise find the equation of the normal to the curve at the point where $\theta = \frac{\pi}{8}$. [4]

(b) **(i)** Evaluate exactly $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (2tan2\theta)^2 \, d\theta$. [6]

(ii) Sketch the graph $f(\theta) = 2tan2\theta$ for $0 \leq \theta \leq \frac{3\pi}{4}$. [4]

7 **(a)** Express $\frac{3+5x}{(2+x)(1+x)^2}$ in partial fractions. [4]

(b) Find the series expansion of $\frac{3+5x}{(2+x)(1+x)^2}$ in ascending powers of x up to and including the terms in x^3. [5]

(c) Using the answer to part (a), find $\sum \frac{3+5x}{(2+x)(1+x)^2} dx$. [4]

(d) Using the answer to part (b) find $f'(x)$ when $x = 0$. [3]

8 Matrix $\text{A} = \begin{pmatrix} -1 & -1 & 1 \\ 0 & -1 & 3 \\ 2 & 1 & 2 \end{pmatrix}$, Matrix $\text{B} = \begin{pmatrix} -3 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix}$ and

$$\text{Matric C} = \begin{pmatrix} 4 & 3 & 1 \\ 2 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix}.$$

(a) Find **AB**. [3]

(b) Show that matrix **C** is singular. [2]

(c) Find the

(i) determinant of **A**, [2]

(ii) inverse of **A**. [5]

(d) Hence or otherwise, solve the simultaneous equations:

$$-x - y + z = 1$$
$$-y + 3z = 8$$
$$2x + y + 2z = 8$$ [4]

9 **(a)** Find the equation of the lines that are mapped onto themselves under the transformation $\begin{pmatrix} 0 & 3 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^1 \\ y^1 \end{pmatrix}$. [11]

(b) The rectangle whose vertices are the points A(0,2), B(0, 4), C(6, 4), D(6, 2) is mapped to the plane figure $A^1B^1C^1D^1$ by $M = \begin{pmatrix} -2 & 1 \\ 0 & -3 \end{pmatrix}$.

Find the area of the rectangle $A^1B^1C^1D^1$. [5]

10 **(a)** The gradient function of a curve is given by $\frac{dy}{dx} = \frac{24}{x^3} - 3$.

Find the equation of the curve given that it passes through the point (1; 16). [5]

(b) A cup of hot coffee at temperature θ°C loses heat when placed in a cooler environment according to the law $\frac{d\theta}{dt} = -k(\theta - \theta_0)$, where t is the time elapsed in minutes, θ_0 is the temperature of the environment in degrees celcius and k is a constant. Initially the temperature of the cup of hot coffee at 100°C is placed in an environment at –20°C. After three minutes the cup of coffee cools to 70°.

(i) Find the particular solution of the differential equation. [8]

(ii) Find the time taken for the cup of coffee to cool to 50°C giving the answer correct to 1 decimal place. [3]

11 Points A and B has position vectors $2\boldsymbol{i} + 2\boldsymbol{j} + \mathbf{k}$ and $\boldsymbol{i} + 4\boldsymbol{j} + 2\mathbf{k}$ respectively. A plane π has equation $x - 2y + 2z = 6$.

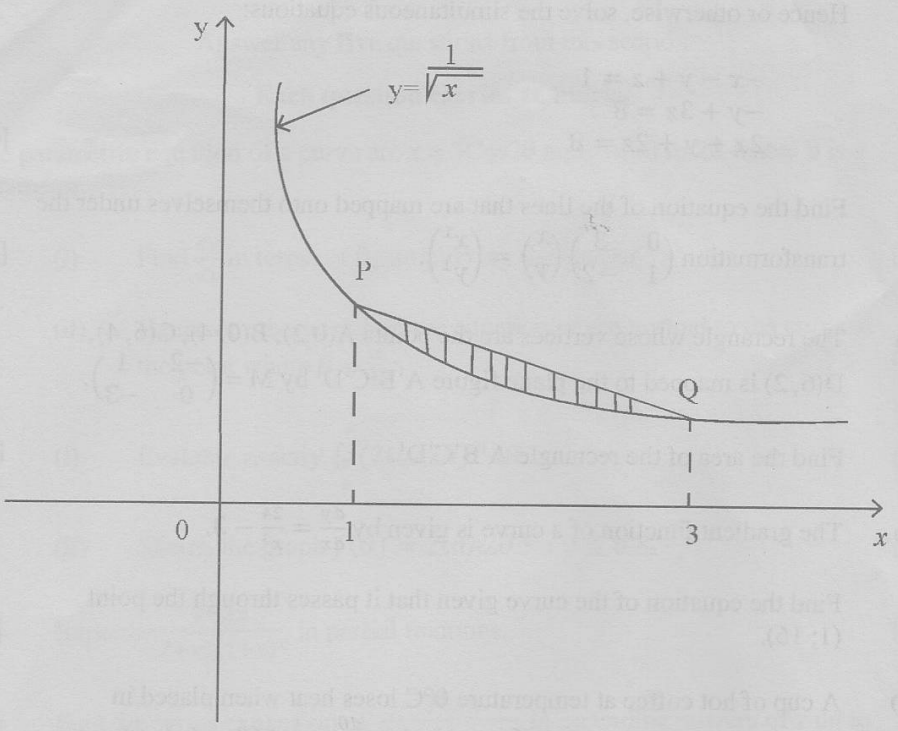
(a) Find the vector equation of a line l, through **A** and **B**. [2]

(b) The line l, which passes through **A** and **B** intersects the plane π at **C**, find the co-ordinates of **C**. [5]

(c) Find the obtuse angle between the line l and plane π correct to the nearest 0,1°. [5]

(d) Find the perpendicular distance from B to plane π. [4]

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The diagram shows the graph of $y = \frac{1}{\sqrt{x}}$. The shaded area is bound by the line **PQ** and the graph of $y = \frac{1}{\sqrt{x}}$.

(a) **(i)** Find the equation of the line **PQ**.

(ii) Calculate the area bound by the line **PQ** and the lines $x = 1$ and $x = 3$ and the x-axis.

(iii) Hence or otherwise, find the shaded area, giving the answer in exact form.

[7]

(b) Using the trapezium rule with five ordinates

find $\int_1^3 \frac{1}{\sqrt{x}}\,dx$ to 3 significant figures. [4]

(c) Find the volume of the solid generated when the area bounded by the curve $y = \frac{1}{\sqrt{x}}$, the lines $x = 1$ and $x = 3$ is rotated completely about the x-axis. [5]