ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/2

PAPER 2

NOVEMBER 2023 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF 7
- Scientific calculator [Non-programmable]

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in Section A and any **five** questions from Section B.

If a numerical answer cannot be given exactly and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

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Section A (40 marks).

Answer **all** questions in this section.

1 (a) Simplify $\frac{3^{2x-3}.81^{x+\frac{1}{2}}}{27^x}$. [3]

(b) Hence or otherwise solve the equation

$$\frac{3^{2x-3}.81^{x+\frac{1}{2}}}{27^x} = \frac{1}{5^{x-1}}$$

giving the answer in exact form. [4]

2 (a) Show that 1 is a root of the polynomial $Q(x) = 2x^3 + 3x^2 - 4x - 1$. [2]

(b) Hence find the polynomial $P(x)$ such that $Q(x) = (x-1)P(x)$ [3]

(c) (i) Express $P(x)$ in the form $A(x+B)^2 + C$. [3]

(ii) Hence state the minimum value of $P(x)$. [1]

3 The curved surface area, A of a cylindrical drum varies jointly as its circular base diameter d, and its height l. Given that the curved surface area of the drum is $2.64m^2$ when its base radius is 0,35m and its height is 1.2m,

(a) find the relationship between A, d and l, [5]

(b) calculate the radius of a cylinder whose surface area is $14{,}52m^2$ and height is 3m to 2 decimal places. [4]

4 Given that $f(x) = kx^2 + 3x + 3$ and $g(x) = kx + 7$, find the range of values of k for which $f(x) = g(x)$ has two distinct solutions. [5]

5 (a) Given that $2x^3 + ax^2 + x - 3 \equiv (bx^2 + c)(x-3)$, find the constants a, b and c. [4]

(b) Hence express $\frac{1+6x}{2x^3+ax^2+x-3}$ in partial fractions. [6]

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Section B (80 marks)

Answer any **five** questions from this section. Each question carries **16** marks.

6 Functions $f(x)$, $t(x)$ and $h(x)$ are defined by

$f(x) = 12x + 20$
$t(x) = e^x$
$h(x) = 2x + 3$.

(a) **(i)** Find $th(x)$. [1]

(ii) On the same axes, sketch the graphs of $y = f(x)$ and $y = th(x)$. [5]

(b) Calculate by integration the area between the graph of $y = f(x)$, $y = th(x)$ and the lines $x = -1.5$ and $x = 0$. [3]

(c) **(i)** Estimate the area in **(b)** using the trapezium rule with 4 ordinates. [4]

(ii) Calculate the relative error made in estimating the area in **(b)** using the trapezium rule. [3]

7 **(a)** Prove the Identity $\frac{2\,Cot\,2\theta}{1-Tan^2\theta} \equiv Cot\,\theta$. [4]

(b) Hence or otherwise solve the equation $\frac{2\,Cot\,2\theta}{1-Tan^2\theta} = 2\,Cosec^2\theta - 1$, for θ in the range $0° \leq \theta \leq 360°$. [12]

8 Matrices A and B^{-1} are given by

$$A = \begin{pmatrix} 1 & 2 & 1 \\ x & 1 & x+1 \\ 2 & x-2 & 0 \end{pmatrix}, x > 0$$

$$B^{-1} = \frac{1}{8}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 3 \end{pmatrix}$$

(a) Calculate the value of x given that the determinant of A is equal to 13 [5]

(b) Hence find

(i) A^{-1}. [7]

(ii) $(AB)^{-1}$. [4]

9 The sum of n terms of a sequence, S_n is given by

$$S_n = \frac{n}{4}(5-n), \text{ for } n = 1; 2;\ 3 \ldots.$$

(a) Find the second and third terms of the sequence. [4]

(b) Hence or otherwise, find the general term of the sequence in term of n. [3]

(c) Calculate the number of terms needed for the Sum, S_n, to be equal to -9. [5]

(d) Find the range of value of n for which $S_n > 0$. [4]

10 A quadrilateral ABCD has vertices at points A(-1;1); B(3;7); C(4;2) and D(2; -1), respectively.

(a) Find the equation of line AB. [4]

(b) Show that line AB is perpendicular to line AD. [4]

(c) Find the equation of a circle with center at point D and radius CD in the form $x^2 + y^2 + ax + by = c$ where a, b and c are constants. [5]

(d) Show that the circle in (c) passes through point A. [2]

(e) Explain why line AB is a tangent to the circle in (c). [1]

11 Triangle ABC has vertices with position vectors $\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$, $2\boldsymbol{i} + 5\boldsymbol{j} - 2\boldsymbol{k}$ and $5\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ respectively.

Find

(a) $\overrightarrow{AC}$, [2]

(b) the cartesian equation of line AC, [4]

(c) (i) the perpendicular distance of point B from line AC, [8]

(ii) the area of triangle ABC. [2]

12 (a) Express $g(x) = \frac{4x+5}{2(x+1)}$ in the form $A + \frac{B}{2(x+1)}$, where A and B are constants. [2]

(b) With the aid of diagrams, in each case, illustrate the transformations which maps the graph of $y = \frac{1}{x}$ on to the graph of $y = g(x)$. [10]

(c) Find the inverse of $g(x)$ and state its domain. [4]

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