ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

CONFIDENTIAL

MARKING SCHEME



NOVEMBER 2015

MATHEMATICS 9164/2

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1 $x^2 + 6y^4 = 7xy^2$

$2x + 24y^3 \frac{dy}{dx} = 7y^2 + 7(2)yx \frac{dy}{dx}$ M1

$2x - 7y^2 = (14xy - 24y^3)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2x - 7y^2}{14xy - 24y^3}$ A1

$= \frac{2-7}{14-24}$ M1

$= \frac{-5}{-10} = \frac{1}{2}$ A1

[4]

2 (i) $R\cos\theta\cos\alpha - R\sin\theta\sin\alpha = 4\cos\theta - 6\sin\alpha$

$R\sin\alpha = 6$

$R\cos\alpha = 4$

$\tan\alpha = \frac{3}{2}$

$\alpha = \tan^{-1}\frac{3}{2}$

$= 56°$

$R^2\cos^2\alpha + R^2\sin^2\alpha = 16 + 36$

$R^2 = 52$

$R = 2\sqrt{13}$

$\therefore\ 4\cos\theta - 6\sin\theta = 2\sqrt{13}\cos(\theta + 56^0)$ B2

(ii) $2\sqrt{13}\cos(\theta + 56) = 5$ M1

$\cos(\theta + 56) = \frac{5}{2\sqrt{13}}$

$\theta + 56° = 46°, 313°, 406° \ldots\ldots$ M1

$\theta = 258°, 350°.$ A1

[6]

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3 $y = mx$

$2(mx) - 3x + 13 = 0$ M1

$x(2m - 3) = -13$

$x = \dfrac{-13}{2m-3}$ A1

$y = mx$ M1

$3y + 2x + 13 = 0$

$3(mx) + 2x + 13 = 0$

$x = \dfrac{-13}{3m+2}$ A1

$\dfrac{1}{2}(x_1 + x_2) = 0$ M1 Idea of M1

$\dfrac{1}{2}\left(\dfrac{-13}{2m-3} + \dfrac{-13}{3m+2}\right) = 0$ M1 equating to zero

$-26m + 39 - 39m - 26 = 0$

$65m = 13$

$m = \dfrac{1}{5}$ A1 answer given

[7]

4 Verify statement true for n = 2

i.e. L.H.S $= 2(2^1)$

$= 4$

R.H.S $= 4(2-1).2^0$

$= 4$

M1 for both.

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L.H.S = R.H.S

$\therefore$ the statement is true for n = 2. A1

Assuming its true for n = k and stating

$$2(2)+3(2^2)+4(2^3)+.....+k(k^{-1}) = 4(k-1).2^{k-2}$$ B1

To prove validity for n = k + 1, i.e.

$$2(2^1)+3(2^2)+.....+k(2^{k-1})+(k+1)2^k = 4(k)(2^{k-1})$$

$$2(2^1)+3(2^2)+.....+k(2^{k-1})+(k+1)2^k$$

$= 4(k-1)2^{k-2}+(k+1)2^k$ M1

$= 4(2^{-2})(k-1)(2^k)+(k+1)2^k$

$= (k-1)(2^k)+(k+1)(2^k)$

$= 2^k(k-1+k+1)$

$= 2^k(2k)$ M1

$= (2k)(2)(2^{k-1})$

$= 4(k)(2^{k-1})$ A1

If true for n=k also true for n = k+1. Since true for n = 2 also true for n = 3 and true for 4 and so on. Therefore by mathematical induction its true for all natural numbers n ≥ 2. A1 [7]

5 (a) $\int_0^{\frac{\pi}{2}} (\sin 2x+\sin x)\,dx$

$= \left[-\frac{1}{2}\cos 2x-\cos x\right]_0^{\frac{\pi}{2}}$ M1

$= -\frac{1}{2}\cos\pi-\cos\frac{\pi}{2}+\frac{1}{2}\cos O+\cos O$ M1

$= +\frac{1}{2}+\frac{1}{2}+1$ A1

$= 2$

[3]

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(b) $\pi\int_0^{\frac{\pi}{2}}(\sin 2x+\sin x)^2\,dx$.

$= \pi\int_0^{\frac{\pi}{2}}\left(\sin^2 2x+2\sin 2x\sin x+\sin^2 x\right)dx$

$= \pi\int\frac{1}{2}(1-\cos 4x)+4\cos x\sin^2 x+\frac{1}{2}(1-\cos 2x)\,dx$

$= \pi\left[\left(\frac{x}{2}-\frac{1}{8}\sin 4x+\frac{4}{3}\sin^3 x+\frac{x}{2}-\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}\right.$

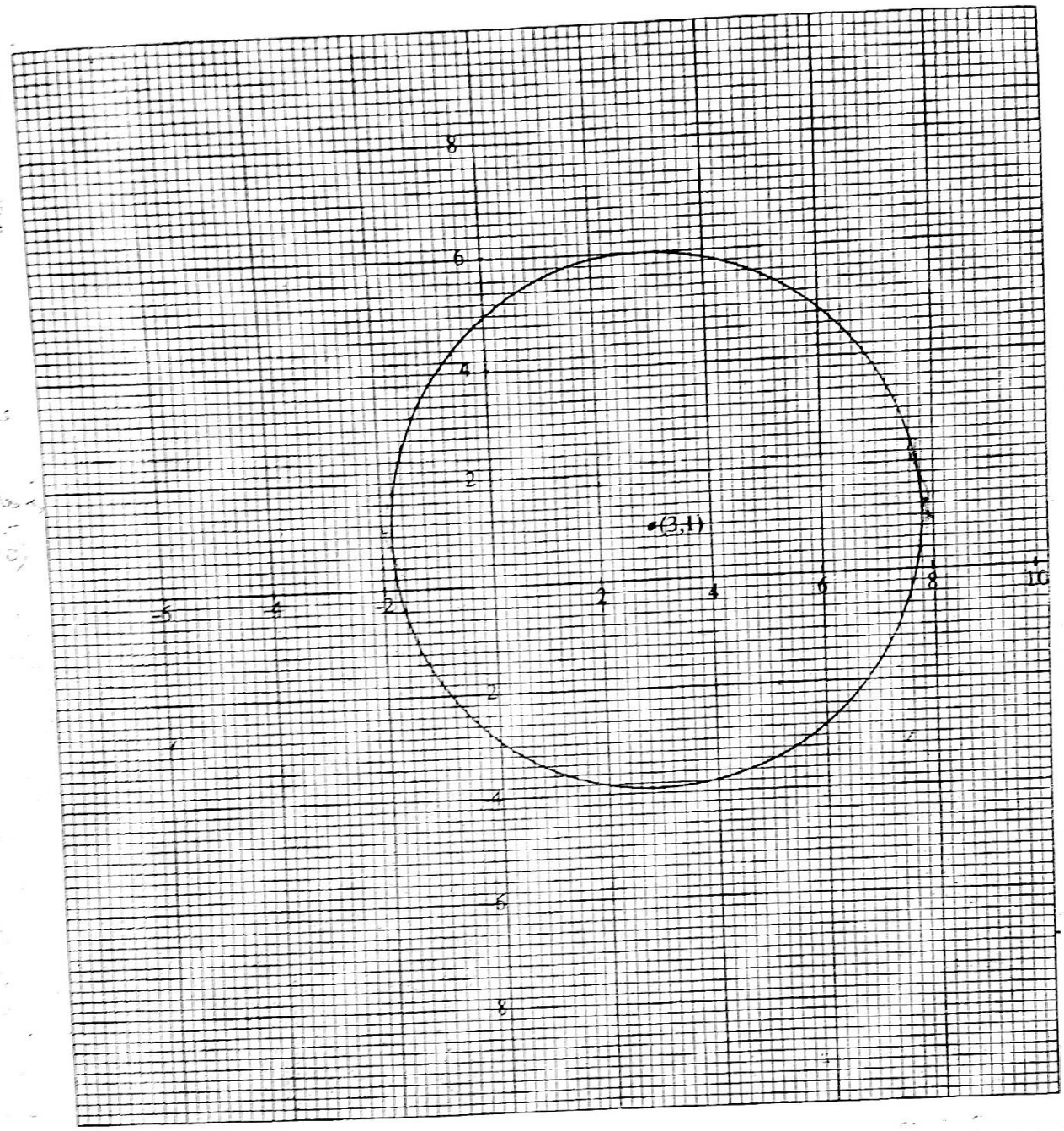
$= \pi\left(\frac{\pi}{4}-0+\frac{4}{3}+\frac{\pi}{4}-0\right)-(0-0+0+0)$ M1

$= \pi\left(\frac{\pi}{2}+\frac{4}{3}\right)$

$= \frac{\pi}{6}(3\pi+8)$ A1 (AG)

[5]

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(i) Z = $-2+i$ *or* $8+i$ B2

(ii) $3 + 6i$ B1

(b) Z = $3\left(Cos\frac{\pi}{2}+iSin\frac{\pi}{2}\right)+4$ M1

= $3(-i) + 4$

= $4 - 3i$

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$|Z| = \sqrt{4^2+3^2}$

$= 5$ A1

(ii) Arg (Z) $= \tan^{-1}\left(\frac{-3}{4}\right)$

$= -36.9°$ B1

7 (a) Det $= 6+5$

$= 11$

Inverse $= \frac{1}{11}\begin{pmatrix} 3 & 1 \\ -5 & 2 \end{pmatrix}$ B1

$\frac{1}{11}\begin{pmatrix} 3 & 1 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} x^1 \\ -14x^1-22 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 3x^1-14x^1-22 \\ -5x^1-28x^1-44 \end{pmatrix}$ M1

$= \frac{1}{11}\begin{pmatrix} -11x^1-22 \\ -33x^1-44 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^1-2 \\ -3x^1-4 \end{pmatrix}$

$x = -x^1-2$..... (1) M1

$y = -3x^1-4$..... (2)

from (1)

$x^1 = -x-2$

Substitute in (2)

$y = -3(-x-2)-4$

$y = 3x+6-4$

$y = 3x+2$ A1

[6]

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$$\theta = 180 - \cos^{-1}\left(\frac{-1}{3\sqrt{3}}\right)$$

$$= 180 - 101.1$$

$$= 78.9^{(0)}$$

A1

[3]

(ii)

$$x - y + z = 4$$
$$2x + y - 2z = 1$$

$$3x - z = 5$$

$$x = \lambda \qquad z = 3\lambda - 5$$

$$y + 2\lambda - 2(3\lambda - 5) = 1$$

$$y + 2\lambda - 6 + 10 = 1$$

$$y = 4\lambda - 9$$

$$\therefore r = \begin{pmatrix} 0 \\ -9 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$$

A1

[4]

(iii)

$$\begin{pmatrix} 0+\lambda \\ -9+4\lambda \\ -5+3\lambda \end{pmatrix} - \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} = \begin{pmatrix} \lambda \\ -3+4\lambda \\ 4+3\lambda \end{pmatrix}$$

$$\begin{pmatrix} \lambda \\ -3+4\lambda \\ 4+3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} = 0$$

$$\lambda - 12 + 16\lambda + 12 + 9\lambda = 0$$

$$26\lambda = 0$$

$$\lambda = 0$$

A1

$$TF = \begin{pmatrix} 0 \\ -3 \\ 4 \end{pmatrix}$$

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$$\overrightarrow{PP'} = \begin{pmatrix} 0 \\ -3 \\ 4 \end{pmatrix}$$

$$|\overrightarrow{PP'}| = \sqrt{(-3)^2 + 4^2 + 0^2}$$

$$= \sqrt{25}$$

$$= 5 \quad \text{A1} \quad [5]$$

(iv) 1.

$$\begin{pmatrix} 3+3t \\ 1+10t \\ 0+5t \end{pmatrix} = \begin{pmatrix} 0+\lambda \\ -9+4\lambda \\ -5+3\lambda \end{pmatrix} \quad \text{M1}$$

$$\lambda = 3 + 3t$$

$$5t = -5 + 3(3 + 3t) \quad \text{M1 for attempt to solve}$$

$$5t = -5 + 9 + 9t$$

$$-4t = 4$$

$$t = -1 \quad \text{A1}$$

Point of intersection

$$= (0, -9, -5) \quad \text{A1}$$

[4]

2.

$$\begin{vmatrix} i & j & k \\ 3 & 10 & 5 \\ 1 & 4 & 3 \end{vmatrix}$$

$$= i\begin{vmatrix} 10 & 5 \\ 4 & 3 \end{vmatrix} - j\begin{vmatrix} 3 & 5 \\ 1 & 3 \end{vmatrix} + k\begin{vmatrix} 3 & 10 \\ 1 & 4 \end{vmatrix}$$

$$= 10i - 4j + 2k \quad \text{A1}$$

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$$\mathbf{r}\begin{pmatrix}5\\-2\\1\end{pmatrix} = \begin{pmatrix}5\\-2\\1\end{pmatrix}\begin{pmatrix}3\\1\\0\end{pmatrix}$$ M1

$$\mathbf{r}\begin{pmatrix}5\\-2\\1\end{pmatrix} = 13$$ A1

[4]

9 Resolve horizontally

$\sqrt{3}\,T_1 = \sqrt{2}\,T_2$

Resolve vertically

$\frac{1}{2}T_1 + \frac{\sqrt{2}}{2}T_2 = 2g$ B1 either one correct

Solve simultaneously

$\frac{1}{2}T_1 + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{\sqrt{2}}T_1 = 2g$ M1

$T_1 = \frac{4g}{1+\sqrt{3}}$

$= 14.36283684$ N A1

$T_2 = \sqrt{\frac{3}{2}}\left(\frac{4g}{1+\sqrt{3}}\right)$

$= 17.59081076$ N A1

[4]

10 (a) particle P: $T - \frac{\sqrt{3}}{5}g - g = 2a$ B1

particle Q: $3g - T = 3a$ B1

$3g + g - \frac{\sqrt{3}}{5}g = 5a$ M1 solve simultaneously or equiv.

$a = \frac{(10-\sqrt{3})}{25}g$

$= 3.244$ A1

[4]

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(b) $T = 3g - \frac{3(10-\sqrt{3})g}{25}$ M1√ substitute his (a)

$= \left(\frac{45+3\sqrt{3}}{25}\right)g$

$= 19.69\text{ N}$ A1 [2]

11 (a) $u = \frac{200}{9}$ B1 s.o.i.

$0 = \left(\frac{200}{9}\right)^2 - 2a\left(\frac{45}{2}\right)$ use of $v^2 = u^2 - 2as$ M1

$a = \frac{8\,000}{729}\ (=10.973)\ \text{ms}^{-2}$ A1 [3]

(b) distance before braking

$\frac{120\times 1\,000}{60\times 60}\times\frac{1}{2} = \frac{50}{3}\text{ m}$

$0 = \left(\frac{100}{3}\right)^2 - 2\left(\frac{8\,000}{729}\right)s$ M1

$s = \frac{405}{8}\text{ m}$ A1

stops in

$\frac{50}{3} + \frac{405}{8} = \frac{1615}{24}\text{m}\ (=67.292\text{ m})$ M1A1 [4]

12 horizontal motion

$15 = (30\cos\alpha)\,t$ or equiv. B1

vertical motion

$5 = (30\sin\alpha)\left(\frac{1}{2\cos\alpha}\right) - \frac{1}{2}g\left(\frac{1}{2\cos\alpha}\right)^2$ or equiv. M1

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$5 = 15\tan\alpha - \frac{9.81}{8}\left(1+\tan^2\alpha\right)$

simplifies to

$1.22625\ \tan^2\alpha - 15\tan\alpha + 6.22625 = 0$ A1

Solving quadratic

$\tan\alpha = \frac{15 \pm \sqrt{15^2 - 4(1.22625)(6.22625)}}{2(1.22625)}$ M1

$= 0.430214$, or 11.802202 A1

$\alpha = 23.278°,\ 85.157°$ ~~for tan~~ A1 [7]

13 (a) Modal age group ~~30 – 35 years~~ 25 – 30 B1

(b) 25 – 30 years = ~~5 x 4.8~~ $5 \times 5,6$

= ~~24 mothers~~ 28 mothers. B1

(c) Mothers in the survey

$= 5 \times 0.8 + 5 \times 3.6 + \cancel{5 \times 4.8} + 5 \times 5.6 + 5 \times 5.2 + \cancel{5 \times 6.0}$ M1

= ~~[illegible]~~ 58 A1 [4]

14 (a) $\int_0^1 ax\,dx + \int_1^2 a(2-x)^2\,dx = 1$

$\left[\frac{ax^2}{2}\right]_0^1 + \left[a\left(4x - \frac{4x^2}{2} + \frac{x^3}{3}\right)\right]_1^2 = 1$ M1 attempt to integrate correct expression

$\frac{a}{2} + \frac{a}{3} = 1$

$\frac{5a}{6} = 1$

$a = \frac{6}{5}$ A1 A5

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(b) (i) $E(X) = \int_0^1 x \times \frac{6}{5}x\,dx + \int_1^2 x \times \frac{6}{5}(2-x)^2\,dx$

$= \left[\frac{6}{5} \times \frac{x^3}{3}\right]_0^1 + \frac{6}{5}\left[\left(\frac{4x^2}{2} - \frac{4x^3}{3} + \frac{x^4}{4}\right)\right]_1^2$ M1

$= \frac{6}{5}\left(\frac{1}{3} + 8 - \frac{32}{3} + 4 - 2 + \frac{4}{3} - \frac{1}{4}\right)$

$= \frac{6}{5} \times \frac{3}{4}$

$= \frac{9}{10}$ or equiv. A1

14 (b) (ii) $\text{Var}(X) = \int_0^1 x^2 \times \frac{5}{6}x\,dx + \int_1^2 x^2 \times \frac{6}{5}(2-x)^2\,dx - \left(\frac{9}{10}\right)^2$

$= \frac{6}{5} \times \frac{x^4}{4}\Big|_0^1 + \frac{6}{5}\left(\frac{4x^3}{3} - \frac{4x^4}{4} + \frac{x^5}{5}\right)_1^2 - \left(\frac{9}{10}\right)^2$ M1

$= \frac{6}{5}\left(\frac{1}{4} + \frac{32}{3} - 16 + \frac{32}{5} - \frac{4}{3} + 1 - \frac{1}{5}\right) - \frac{81}{100}$

$= \frac{47}{50} - \frac{81}{100}$

$= \frac{13}{100}$

$= 0.13$ A1 [6]

15 $X \sim N\left(5, \left(\frac{2}{3}\right)^2\right)$

(i) 1. $P(X > 6) = P\left(Z > \frac{6-5}{\frac{2}{3}}\right)$ M1

$= P(Z > 1{,}5)$

$= 1 - \phi(1{,}5)$

$= 1 - 0.9332$

$= 0.0668$ A1

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2. $P\left(4\frac{1}{2} < X < 5\right) = P\left(\frac{4\frac{1}{2}-5}{\frac{2}{3}} < Z < \frac{5-5}{\frac{2}{3}}\right)$ M1

$= P(-0.75 < Z < 0)$

$= \phi(0) - (1 - \phi(0.75))$

$= 0.5 - (1 - 0.7734)$

$= 0.2734$ A1

$= 27.34\ \%$ B1

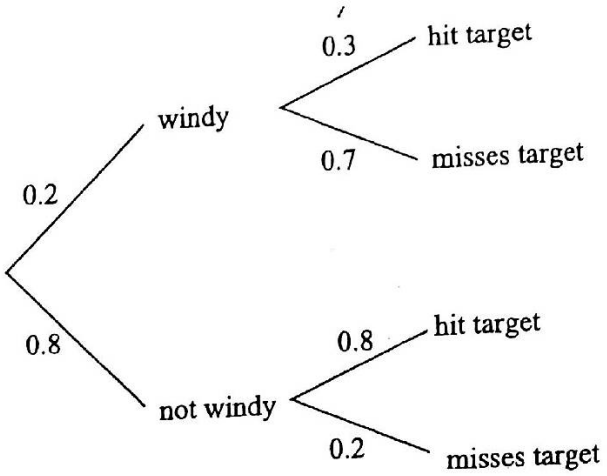
(ii) $X \sim \text{Bin}(6, 0.2734)$

$P(X = 4) = {}^{6}C_{4} \times 0.2734^{4} \times 0.7266^{2}$ M1

0.0442 A1

[7]

16 (a)



B1

B1

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