This is chapter 1 of the book, Exam Cheat Code: Maths 4. The book is designed to prepare students for Zimsec/Cambridge Exams. The book come with the following:

- Step by step worked examples.
- Detailed answers.
- Exam type Questions.
- Your Name, Nickname and Surname.
- A password only for you, derived from your birth date and birth month.

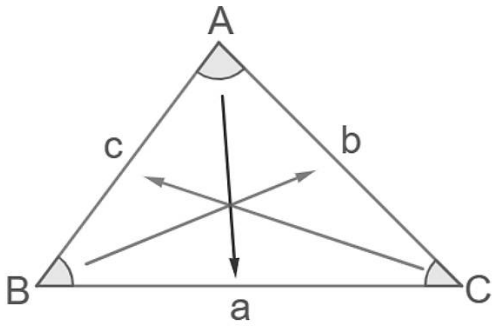
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1

Trigonometry Two

Sine Rule

The Sine Rule states that for any triangle (not just right-angled triangles), the ratio of the length of a side to the sine of its opposite angle is constant.



The ratio can be written as:

When looking for sides.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

When looking for angles.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

where:

a, b, c = sides of the triangle

A, B, C = angles opposite sides a, b, c

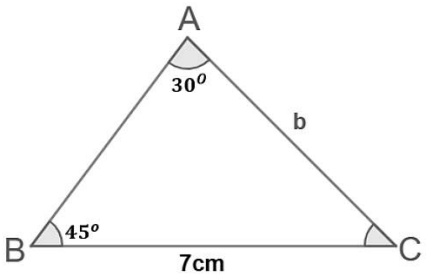
When to Use the Sine Rule

- When you know two angles and one side (AAS or ASA cases).
- When you know two sides and an angle not included between them (SSA case).
- Can apply to all triangles right-angled and non-right angled. The Sine Rule is particularly useful for non-right-angled triangles.

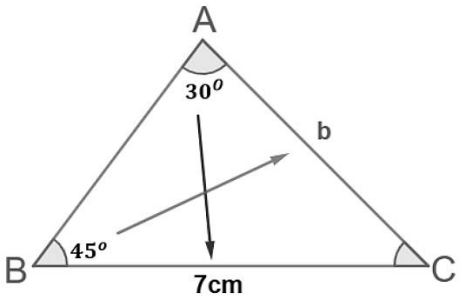
Example 1

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Given triangle ABC, with A= 30^o, B= 45^o and $a = 7cm$, find b



Solution:



We are looking for a side b, we have information for sides and angles of A and B, we have nothing on C.

When looking for sides.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{7}{\sin 30^o} = \frac{b}{\sin 45^o}$$

$\sin 45^o \times \frac{7}{\sin 30^o} = \frac{b}{\sin 45^o} \times \sin 45^o$ (multiply both sides by $\sin 45^o$ to isolate b)

$\sin 45^o \times \frac{7}{\sin 30^o} = \frac{b}{\cancel{\sin 45^o}} \times \cancel{\sin 45^o}$ ($\sin 45^o$ cancels out on the RHS)

$\frac{\sin 45^o \times 7}{\sin 30^o} = b$

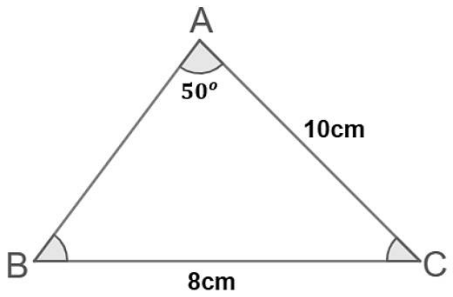
$\frac{7 \sin 45^o}{\sin 30^o} = b$ (use calculator in degrees mode)

9.90cm=b

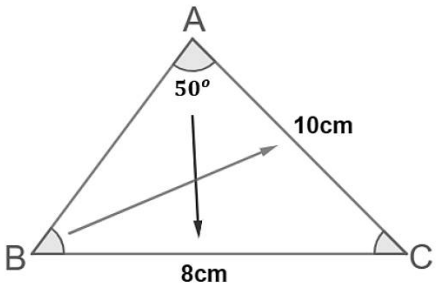
$\therefore \boldsymbol{b = 9.90cm}$ (to one decimal place)

Example 2

Given triangle ABC with sides a =8cm, b=10cm and angle $B\hat{A}C = 50^o$, find B.



Solution:



We are looking for angle B, we have information for sides and angles of A and B, we have nothing on C.

When looking for angles.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 50^o}{8} = \frac{\sin B}{10}$$

$10 \times \frac{\sin 50^o}{8} = \frac{\sin B}{10} \times 10$ (multiply both sides by 10 to isolate sinB)

$10 \times \frac{\sin 50^o}{8} = \frac{\sin B}{\cancel{10}} \times \cancel{10}$ (10 cancels out on the RHS)

$$10 \times \frac{\sin 50^o}{8} = \sin B$$

$$\frac{10 \times \sin 50^o}{8} = \sin B$$

$$0.9576 = \sin B$$

$$\sin^{-1} 0.9576 = B$$

$$73.2^o = B$$

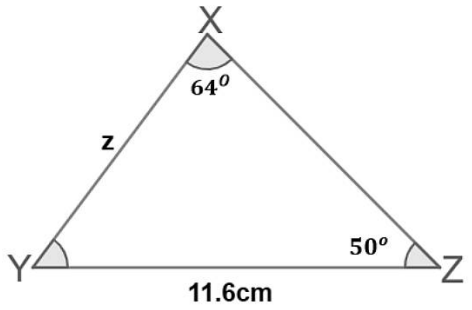
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$\therefore \boldsymbol{B = 73.2^o}$ (to one decimal place)

Exercise 1a [30 marks] [Answers]

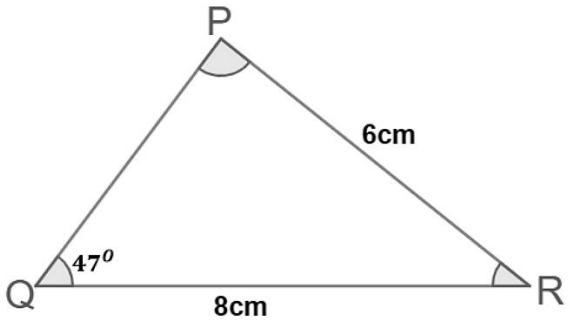
Leave lengths to 3 significant figures and angles to 1 decimal place.

1. Given that $Y\hat{X}Z = 64^o$, $X\hat{Z}Y = 50^o$,$x = 11.6cm$. Find z



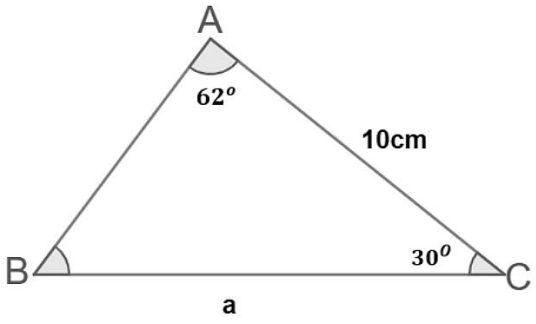
[3]

2. Given that $P\hat{Q}R = 47^o$, q=6cm, p=8cm. Find angle P.



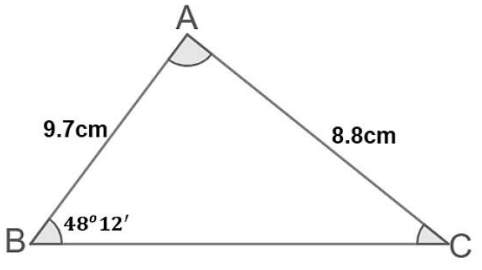
[3]

3. Given that $B\hat{A}C = 62^o$, $A\hat{C}B = 30^o$, b=10cm. Find a



[3]

4. Given that $A\hat{B}C = 48^o12^o$, c=9.7cm and b=8.8cm. Find angle C.



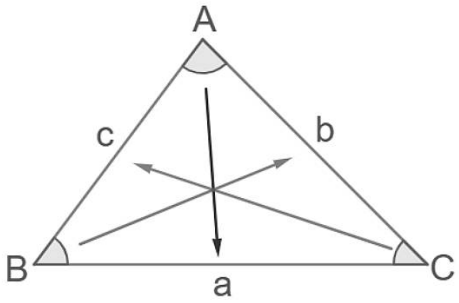
[3]

5. In ΔABC, A = 40°, B = 60°, a = 15 cm. Find the length of side b. [3]

6. In ΔPQR, p = 12 cm, Q = $45^o36'$, R = $60^o12'$. Calculate the length of side q. [3]
7. In ΔMNO, m = 8 cm, N = 53°, O = 47°. Calculate the length of side n. [3]
8. In ΔXYZ, x = 10 cm, y = 12 cm, X=49^o cm. Find angle Y. [3]
9. In ΔEFG, e = 18 cm, f = 12 cm, E = $67^o19'$. Calculate angle F. [3]
10. In ΔJKL, j = 20 cm, k = 24 cm, J = 45°. Find angle K. [3]

Cosine Rule

The Cosine Rule is useful for finding the lengths of sides or angles in any triangle, not just right-angled triangles. The Cosine Rule is particularly useful for non-right-angled triangles.



Finding sides:

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Finding Angles:

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos B = \frac{a^2+c^2-b^2}{2ac}$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

where:

a, b, c = sides of the triangle

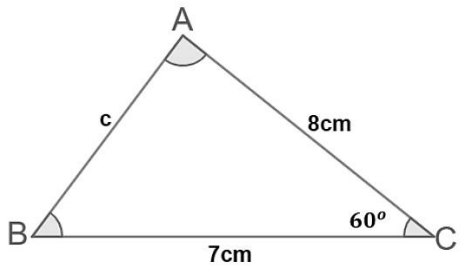
A, B, C = angles opposite sides a, b, c

When to Use the Cosine Rule

- When you know two sides and the included angle (SAS case).
- When you know all three sides and need to find an angle (SSS case).

Example 3

Given that $A\hat{C}B = 60^o$,b=8cm and a=7cm. Find c



Solution:

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 7^2 + 8^2 - 2 \times 7 \times 8 \times \cos 60^o$

$c^2 = 49 + 64 - 112 \times \cos 60^o$

$c^2 = 113 - 112\cos 60^o$ (use a calculator in degree mode)

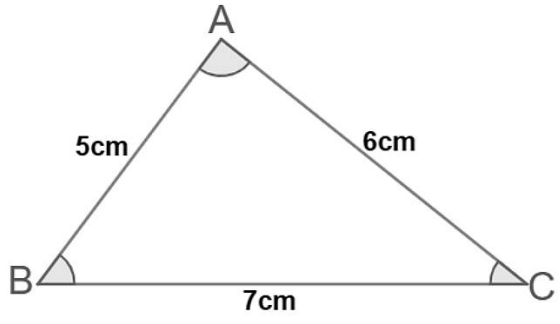
$c^2 = 57$

$\sqrt{c^2} = \sqrt{57}$

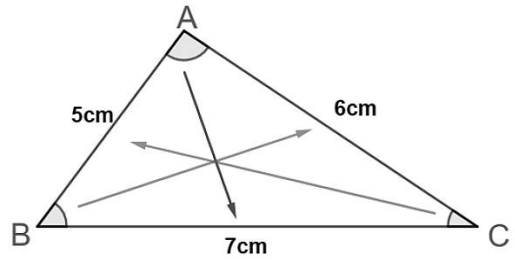
$\boldsymbol{c = 7.55cm}$ (to 3 significant figures)

Example 4

Given that triangle ABC sides AC=6cm, AB=5cm and BC=7cm. Find all its angles.



Solution:



Finding angle, A

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{6^2+5^2-7^2}{2\times6\times5}$$

$$= \frac{36+25-49}{60}$$

$$\cos A = \frac{12}{60}$$

$$A = \cos^{-1}\left(\frac{12}{60}\right)$$

$$\boldsymbol{A = 78.5^o}$$

Finding angle B

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{7^2+5^2-6^2}{2\times7\times5}$$

$$= \frac{49+25-36}{70}$$

$$\cos B = \frac{38}{70}$$

$$B = \cos^{-1}\left(\frac{38}{70}\right)$$

$$\boldsymbol{B = 57.1^o}$$

Finding angle C

Angles in triangle add up to 180^o, since we now have two of the angles so to find the third, we add the two and subtract from 180^o

$$C = 180^o - (78.5^o + 57.1^o)$$

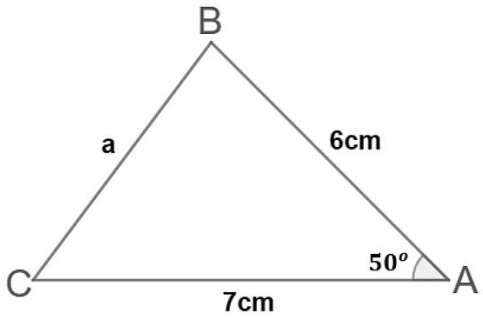
$$= 180^o - 135.6^o$$

$$\boldsymbol{= 44.4^o}$$

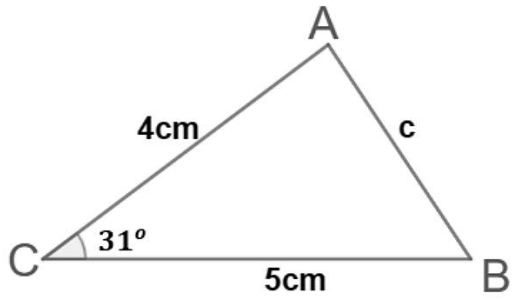
Exercise 1b [30 marks] [Answers]

Leave lengths to 3 significant figures and angles to 1 decimal place.

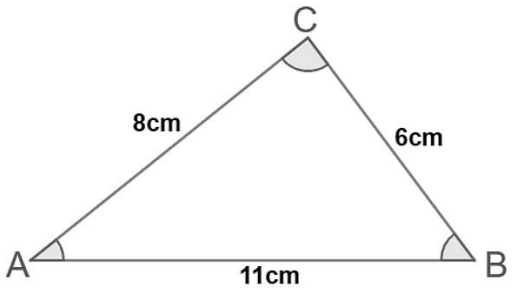
1. Find a



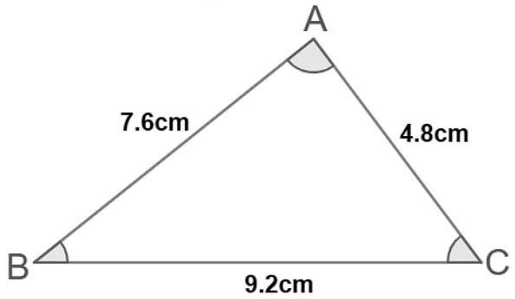
2. Find c



3. Find all three angles.



4. Find all three angles.



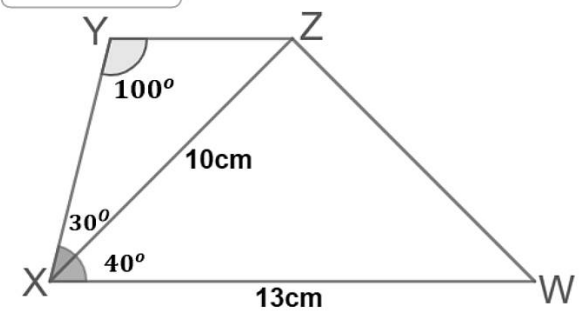
5. Given that $\hat{B} = 132^o$, a=5cm and c=5.9cm. Find b

6. Given that $\hat{C} = 33^o12'$, b=4.5cm and a=6cm. Find c

7. Given that $\hat{A} = 142{,}2^o$, b=42cm and c=22cm. Find a

8. Given that a=4.5cm, b=3.3cm and c=2.2cm. Find all three angles.
9. Given that a=5.8cm, b=4.1cm, and c=7.2cm, find all three angles of the triangle.
10. Given that a=6.2cm, b=3.5cm, and c=5.9cm, find all three angles of the triangle.

Example 5



(a) Find the length of WZ
(b) Find the length of XY
(c) Find angle $Z\hat{W}X$
(d) Find the shortest distance from Z to XW.

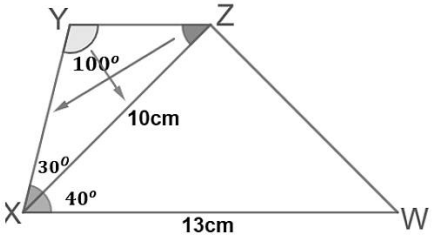
Solution.

(a) Find the length of WZ
We will use triangle WXZ, we two sides and an angle and we are looking for the side opposite the given angle so we use the cosine rule.

$$\begin{aligned}(WZ)^2 &= 10^2 + 13^2 - 2(10)(13)\cos 40^o \\ &= 100 + 169 - 260\cos 40^o \\ &= 269 - 260\cos 40^o \\ (WZ)^2 &= 69.8284448 \quad \text{(use a calculator in degrees mode)} \\ \sqrt{(WZ)^2} &= \sqrt{69.8284448} \quad \text{(use calculator)}\end{aligned}$$

WZ=8.36cm

(b) Find the length of XY
We will use triangle XYZ, we have two angles and a side so we will use the sine rule to find XY.



We first find the angle $Y\hat{Z}X$ (angle in blue) which is opposite XY.

$Y\hat{Z}X = 180^o - (100^o + 30^o)$ (angles in a tringle add up to 180^o)
$= 180^o - 130^o$
$= 50^o$

Now we find XY using the sine rule.

$$\frac{XY}{\sin 50^o} = \frac{10}{\sin 100^o}$$

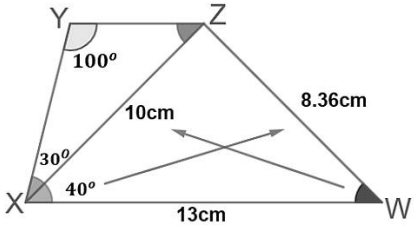
$\sin 50^o \times \frac{XY}{\sin 50^o} = \frac{10}{\sin 100^o} \times \sin 50^o$ (multiply both sides by $\sin 50^o$ to isolate XY)

$XY = \frac{10 \times \sin 50^o}{\sin 100^o}$ (use a calculator in degrees mode)

$\boldsymbol{XY = 7.78cm}$ (to 3 significant figures)

(c) Find angle $Z\widehat{W}X$

We are going to use triangle WXZ, to find angle W we can use either the cosine rule or the sine rule. We are going to use the sine rule.



$$\frac{\sin \widehat{W}}{10} = \frac{\sin 40^o}{8.36}$$

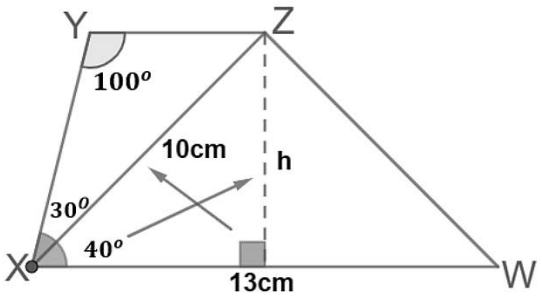
$10 \times \frac{\sin \widehat{W}}{10} = \frac{\sin 40^o}{8.36} \times 10$ (multiply both sides by 10 to isolate $\sin \widehat{W}$)

$\sin \widehat{W} = \frac{10 \times \sin 40^o}{8.36}$ (use a calculator in degrees mode)

$\boldsymbol{\sin \widehat{W} = 50.3^o}$ (to one decimal place)

(d) Find the shortest distance from Z to XW.

The shortest distance from Z to XW is perpendicular to XW. To find h (the shortest distance from Z to XW) we use trig ratios. In the right-angled triangle h is the opposite, and 10cm is the hypotenuse. So, we use Sine.



$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\sin 40^o = \frac{h}{10}$$

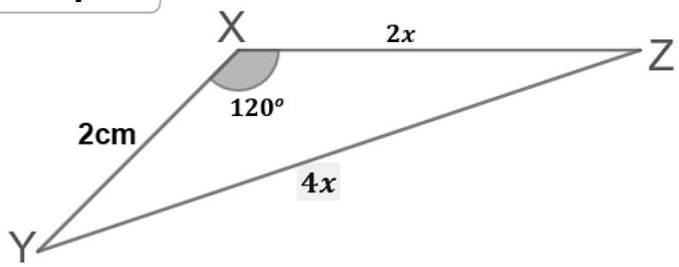
$10 \times \sin 40^o = \frac{h}{10} \times 10$ (multiply both sides by 10 to isolate h)

$10 \sin 40^o = h$ (use a calculator in degrees mode)

6.43cm=h

$\therefore \boldsymbol{h = 6.43cm}$ (to 3 significant figures)

Example 6



In the diagram, XYZ is a triangle in which YX = 2 cm, XZ = 2x cm, YZ = 4x cm and $Y\hat{X}Z = 120^o$.

(i) Form an equation in x and show that it reduces to $3x^2 - x - 1 = 0$.

(ii) Hence find the value of x to 3 significant figures.

(iii) Find YZ.

Solution:

(i) Form an equation in x and show that it reduces to $3x^2 - x - 1 = 0$.

We use the cosine rule to create the equation.

$$(4x)^2 = 2^2 + (2x)^2 - 2 \times 2x \times 2 \times \cos 120^o$$

$16x^2 = 4 + 4x^2 - 8x \times (-0.5)$ $(\cos 120^o = -0.5)$

$16x^2 = 4 + 4x^2 + 4x$ $[-8x \times (-0.5) = 4x]$

$16x^2 - 4x^2 - 4x - 4 = 0$ (collect everything on the left side)

$12x^2 - 4x - 4 = 0$ $(16x^2 - 4x^2 = 12x^2)$

$4(3x^2 - x - 1) = 0$ (factor out 4)

$\frac{4(3x^2-x-1)}{4} = \frac{0}{4}$ (divide both sides by 4)

$\mathbf{3x^2 - x - 1 = 0}$

(ii) Hence find the value of x

To find x we solve $3x^2 - x - 1 = 0$ using the quadratic equation.

$a = 3, b = -1, c = -1$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-1) \pm \sqrt{(-1)^2-4\times3\times(-1)}}{2\times3}$

$= \frac{1 \pm \sqrt{1+12}}{6}$

$= \frac{1 \pm \sqrt{13}}{6}$

$= \frac{1+\sqrt{13}}{6}$ or $\frac{1-\sqrt{13}}{6}$

$= 0.768\text{cm}$ or -0.434cm

We discard the value -0.434cm because length cannot be a negative.

$\therefore \boldsymbol{x = 0.768cm}$

(iii) Find YZ.

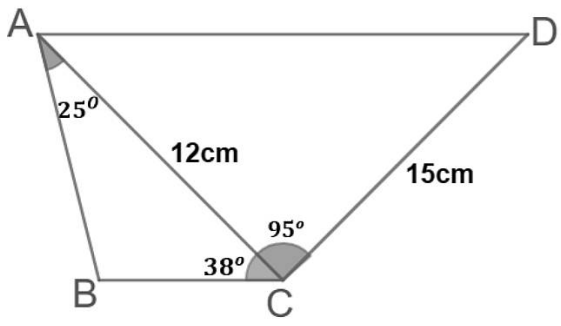
$YZ = 4x$

$= 4 \times 0.768$

$= \mathbf{3.072cm}$

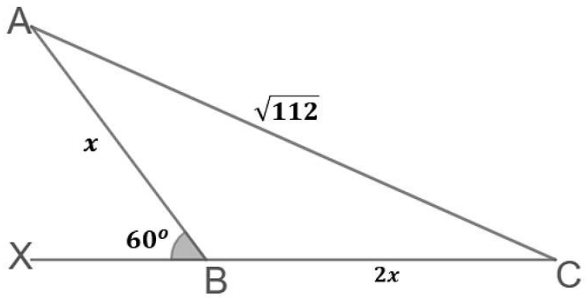
Exercise 1c [30 marks] [Answers]

1.



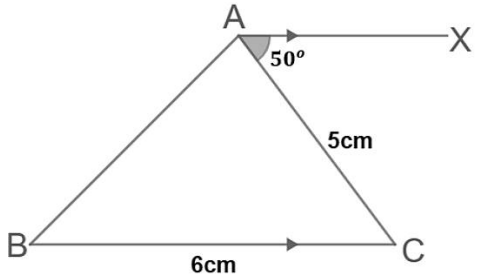
(i) Find AD. [3]
(ii) Find AB. [3]
(iii) Find the shortest distance from B to AC. [2]
(iv) Find the area of triangle ACD. [3]

2.



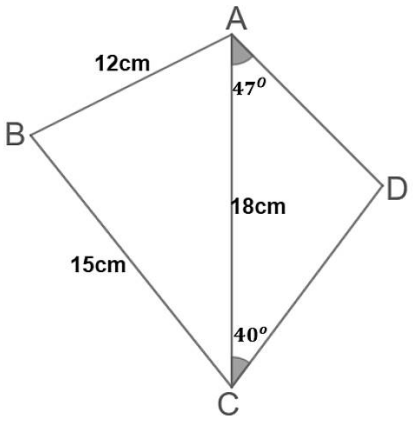
In the diagram, ABC is a triangle in which AB = $x$cm, BC=2x cm, AC= $\sqrt{112}$ cm and $A\hat{B}X = 60^o$
(i) Form an equation in x and show that it reduces to $7x^2 - 112 = 0$ [3]
(ii) Hence find the value of x. [3]
(iii) Find BC. [1]

3. AX is parallel to BC, $X\hat{A}C = 50^o$



(i) Find AB. [3]
(ii) Find the shortest distance from A to BC. [3]

4.



(i) Find AD. [3]

(ii) Find $A\hat{B}C$ to 1 decimal place. [3]

Answers maths book 4

Exercise 1a back to [Trigonometry]

1. 9.89cm $\left[\frac{z}{\sin 50^o}=\frac{11.6}{\sin 64^o}\right]\left[z=\frac{11.6\sin 50^o}{\sin 64^o}\right]$

2. 77.2^o $\left[\frac{\sin P}{8}=\frac{\sin 47^o}{6}\right]\left[\sin P=\frac{8\sin 47^o}{6}\right][\sin P=0.9751..][P=\sin^{-1}(0.9751\ldots)]$

3. 8.83cm $\left[\hat{B}=180^o-(62^o+30^o)=88^o\right]\left[\frac{a}{\sin 62^o}=\frac{10}{\sin 88^o}\right]\left[a=\frac{10\sin 62^o}{\sin 88^o}\right]$

4. 55.3^o $\left[\frac{\sin C}{9.7}=\frac{\sin 48^o12'}{8.8}\right]\left[\sin C=\frac{9.7\sin 48^o12'}{8.8}\right][\sin C=0.8217..][C=\sin^{-1}(0.8217\ldots)]$

5. 20.2cm $\left[\frac{b}{\sin 60^o}=\frac{15}{\sin 40^o}\right]\left[b=\frac{15\sin 60^o}{\sin 40^o}\right]$

6. 8.91cm $\left[\hat{P}=180^o-(45^o36'+30^o12')=74^o12'\right]\left[\frac{q}{\sin 45^o36'}=\frac{12}{\sin 74^o12'}\right]\left[q=\frac{12\sin 45^o36'}{\sin 74^o12'}\right]$

7. 6.49cm $\left[\hat{M}=180^o-(53^o+47^o)=80^o\right]\left[\frac{n}{\sin 53^o}=\frac{8}{\sin 80^o}\right]\left[n=\frac{8\sin 53^o}{\sin 80^o}\right]$

8. 64.9^o $\left[\frac{\sin Y}{12}=\frac{\sin 49^o}{10}\right]\left[\sin Y=\frac{12\sin 49^o}{10}\right][\sin Y=0.9057..][Y=\sin^{-1}(0.9057\ldots)]$

9. 38^o $\left[\frac{\sin F}{12}=\frac{\sin 67^o19'}{18}\right]\left[\sin Y=\frac{12\sin 67^o19'}{18}\right][\sin Y=0.6151..][Y=\sin^{-1}(0.6151\ldots)]$

10. 58.1^o $\left[\frac{\sin K}{24}=\frac{\sin 45^o}{20}\right]\left[\sin K=\frac{24\sin 45^o}{20}\right][\sin K=0.8485..][Y=\sin^{-1}(0.8485\ldots)]$

Exercise 1b [Trigonometry]

1. 5.57cm

$[a^2=6^2+7^2-2(6)(7)\cos 50^o][a^2=36+49-84\cos 50^o][a^2=85-84\cos 50^o]\left[a=\sqrt{85-84\cos 50^o}\right]$

2. 2.59cm

$[c^2=4^2+5^2-2(4)(5)\cos 31^o][c^2=16+25-40\cos 31^o][c^2=41-40\cos 31^o]\left[c=\sqrt{41-40\cos 31^o}\right]$

3. $\hat{A}=32.2^o;\ \hat{B}=45.2^o;\ \hat{C}=102.6^o$

$\left[cosA=\frac{8^2+11^2-6^2}{2\times8\times11}=\frac{64+121-36}{176}=\frac{149}{176}\right]\left[A=\cos^{-1}\left(\frac{149}{176}\right)\right]$

$\left[cosB=\frac{6^2+11^2-8^2}{2\times6\times11}=\frac{36+121-64}{132}=\frac{93}{132}\right]\left[B=\cos^{-1}\left(\frac{93}{132}\right)\right]$

$[C=180^o-(45.2^o+32.2^o)]$

4. $\hat{A}=93^o;\ \hat{B}=31.4^o;\ \hat{C}=55.6^o$

$\left[cosA=\frac{7.6^2+14.8^2-9.2^2}{2\times7.6\times4.8}=\frac{57.76+23.04-84.64}{72.96}=\frac{-3.84}{72.96}\right]\left[A=\cos^{-1}\left(\frac{-3.84}{72.96}\right)\right]$

$\left[cosB=\frac{7.6^2+9.2^2-4.8^2}{2\times7.6\times9.2}=\frac{57.76+84.64-23.04}{139.84}=\frac{119.36}{139.84}\right]\left[B=\cos^{-1}\left(\frac{119.36}{139.84}\right)\right]$

$[C=180^o-(31.4^o+93^o)]$

5. 9.96cm

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$[b^2 = 5^2 + 5.9^2 - 2(5)(5.9)\cos 132^o][b^2 = 25 + 34.81 - 59\cos 132^o][b^2 = 59.81 - 59\cos 132^o]\left[b = \sqrt{59.81 - 59\cos 132^o}\right]$

6. 3.33cm

$[c^2 = 6^2 + 4.5^2 - 2(6)(4.5)\cos 33^o 12'][c^2 = 36 + 20.25 - 54\cos 33^o 12'][c^2 = 56.25 - 54\cos 33^o 12']\left[c = \sqrt{56.25 - 54\cos 33^o 12'}\right]$

7. 60.9cm

$[a^2 = 42^2 + 22^2 - 2(42)(22)\cos 142.2^o][a^2 = 1764 + 484 - 1848\cos 142.2^o][a^2 = 2248 - 1848\cos 142.2^o]\left[a = \sqrt{2248 - 1848\cos 142.2^o}\right]$

8. $\hat{A} = 108.1^o; \hat{B} = 44.2^o; \hat{C} = 27.7^o$

$\left[cosA = \frac{3.3^2+2.2^2-4.5^2}{2\times3.3\times2.2} = \frac{10.89+4.84-20.25}{14.52} = \frac{-4.52}{14.52}\right]\left[A = \cos^{-1}\left(\frac{-4.52}{14.52}\right)\right]$

$\left[cosB = \frac{4.5^2+2.2^2-3.3^2}{2\times4.5\times2.2} = \frac{20.25+4.84-10.89}{19.8} = \frac{14.2}{19.8}\right]\left[B = \cos^{-1}\left(\frac{14.2}{19.8}\right)\right]$

$[C = 180^o - (108.1^o + 44.2^o)]$

9. $\hat{A} = 53.6^o; \hat{B} = 34.7^o; \hat{C} = 91.7^o$

$\left[cosA = \frac{4.1^2+7.2^2-5.8^2}{2\times4.1\times7.2} = \frac{16.81+51.84-33.64}{59.04} = \frac{35.01}{59.04}\right]\left[A = \cos^{-1}\left(\frac{35.01}{59.04}\right)\right]$

$\left[cosB = \frac{7.2^2+5.8^2-4.1^2}{2\times7.2\times5.8} = \frac{51.84+33.64-16.81}{83.52} = \frac{68.67}{83.52}\right]\left[B = \cos^{-1}\left(\frac{68.67}{83.52}\right)\right]$

$[C = 180^o - (53.7^o + 34.7^o)]$

10. $\hat{A} = 78^o; \hat{B} = 33.5^o; \hat{C} = 68.5^o$

$\left[cosA = \frac{3.5^2+5.9^2-6.2^2}{2\times3.5\times5.9} = \frac{12.25+34.81-38.44}{41.3} = \frac{8.62}{41.3}\right]\left[A = \cos^{-1}\left(\frac{8.62}{41.3}\right)\right]$

$\left[cosB = \frac{6.2^2+5.9^2-3.5^2}{2\times6.2\times5.9} = \frac{38.44+34.81-12.25}{73.16} = \frac{61}{73.16}\right]\left[B = \cos^{-1}\left(\frac{61}{73.16}\right)\right]$

$[C = 180^o - (78^o + 33.5^o)]$

Exercise 1c [Trigonometry]

1. (i) 20 cm (cosine rule)

$[AD^2 = 12^2 + 15^2 - 2(12)(15)\cos 95^o][AD^2 = 144 + 225 - 360\cos 95^o][AD^2 = 369 - 360\cos 95^o]\left[AD = \sqrt{369 - 360\cos 95^o}\right]$

(ii) 8.29cm (sine rule)

$\left[\hat{B} = 180^o - (25^o + 38^o)\right]\left[\frac{AB}{\sin 38^o} = \frac{12}{\sin 117^o}\right]\left[AB = \frac{12\sin 38^o}{\sin 117^o}\right]$

(iii) 3.5cm $\left[\sin 25^o = \frac{x}{8.29}\right][x = 8.29\sin 25^o]$

(iv) $89.7cm^2$ $\left[area = \frac{1}{2}\times 12\times 15\times \sin 95^o = 90\sin 95^o\right]$

2. (i) $\left[\left(\sqrt{112}\right)^2 = x^2 + (2x)^2 - 2(x)(2x)\cos(180^o - 60^o)\right]\left[112 = x^2 + 4x^2 - 4x^2\times\left(-\frac{1}{2}\right)\right][112 = x^2 + 4x^2 + 2x^2][112 = 7x^2][0 = 7x^2 - 112]$

(ii) 4cm $[7x^2 = 112]\left[x^2 = \frac{112}{7}\right]\left[x = \sqrt{\frac{112}{7}} = \sqrt{16} = \pm 4\right]$ [−4 is discarded length cannot be negative]

(iii) 8cm. $[2x = 2 \times 4 = 8]$

3. (i) 4.74cm (cosine rule)

$[AB^2 = 5^2 + 6^2 - 2(5)(6)\cos 50^o][AB^2 = 25 + 36 - 60\cos 50^o][AB^2 = 61 - 60\cos 50^o]\left[AB = \sqrt{61 - 60\cos 50^o}\right]$

(ii) 3.83cm. $\left[\sin 50^o = \frac{x}{5}\right][x = 5\sin 50^o]$

4. (i) 11.6cm (sine rule)

$\left[A\hat{B}C = 180^o - (47^o + 40^o)\right]\left[\frac{AD}{\sin 40^o} = \frac{18}{\sin 93^o}\right]\left[AD = \frac{18\sin 40^o}{\sin 93^o}\right]$

(ii) 82.8^o (cosine rule)

$\left[cosA\hat{B}C = \frac{12^2+15^2-18^2}{2\times 15\times 12} = \frac{144+225-324}{360} = \frac{45}{360}\right]\left[A = \cos^{-1}\left(\frac{45}{360}\right)\right]$